



About This Book

This book covers all you need to know for Standard Grade Maths at General and Credit Levels. This new revised edition includes the Statistics and non-calculator content added to the syllabus for the 2001 exams.

The first seven chapters cover the seven main topic areas of the syllabus. Chapter 8 gives examples on problem solving. Each chapter is divided into sections to make it easier for you to find a specific topic. All key results are 'boxed' and key terms are shown in **bold like this** so that you can find them quickly.

The main difference between the levels in Standard Grade Maths is the content included in each. All the material in each topic has been grouped together regardless of its level in the syllabus. Consequently, each section has been clearly labelled with the level so that you know exactly which parts you require to study.

The Scottish Qualifications Authority has identified the 'easier' Credit material to allow pupils to attempt Credit Level, without having covered all the topics. Therefore, to make this book useful to as many pupils as possible, the material has been labelled as follows:-

- G** All the material required for the **General** Level. If you are trying for General Level and not attempting Credit, this is the work you should concentrate on.
- C/G** The '**easier**' **Credit** Level material. If you are hoping to extend from General Level by trying some Credit Level work you should concentrate on this material, as well as the **G** topics. Check with your teacher if you are not sure if you have covered all these topics - your teacher is the best judge of what is appropriate for you.
- C** The '**harder**' **Credit** Level material. If you are covering the whole Credit course you need to be familiar with all the topics (**G**, **C/G** and **C**) in the book.

The **Contents** on pages 2 and 3 gives the general outline of the book. A **Detailed Index** has been added on pages 150 to 154 to help you find particular topics or terms easily.

The book can be used:

as a reference book - when you come across a term or topic that you are unsure about, look up that topic area in the book and refresh your memory, but make sure you understand it properly when you do. Keep the book for reference if you intend to study maths after Standard Grade.

to revise a particular topic - read through the material carefully. Make sure that you know all formulae and standard results. Study the examples given so that you understand how they are worked out. Then try some more examples on the topic from your textbook or jotters.

alongside past papers - in the final lead-up to your exams, the best revision is to work through exam papers from previous years. When you come across a question that you are unable to answer, look up that topic area in the book and see if a similar example is given. With over 200 examples in the book, there is a good chance of finding something that will help.

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5. TRIGONOMETRY

Trigonometry literally means the “measure of triangles”.

Trigonometry looks at the relationships between angles and sides, firstly in the right angled triangle, and then in other triangles.

However, trigonometry goes beyond triangles to include the relationships between an angle and a number value produced by a trigonometric function.

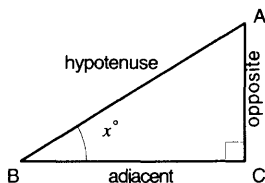
5.1 THE RIGHT ANGLED TRIANGLE

5.1.1 Naming the Sides

The longest side (opposite the right angle) is called the **hypotenuse** (hyp).

In trigonometry, you are always interested in a particular angle.

The angle, if it is unknown, may be represented by a letter, eg x .



You may prefer to use a Greek letter like “theta” (θ) for angles.

The position of this angle determines the names of the other sides.

The side opposite the angle is called the **opposite** (opp), and the side next to the angle is called the **adjacent** (adj).

5.1.2 The Trigonometric Ratios in the Right Angled Triangle

You can define 3 ratios in the right angled triangle which will always give the same values for a given angle x° , regardless of the size of triangle.

You call these ratios the **sine of x°** ($\sin x^\circ$), the **cosine of x°** ($\cos x^\circ$) and the **tangent of x°** ($\tan x^\circ$). They are defined to be:

$\sin x^\circ = \frac{\text{opp}}{\text{hyp}}$	$\cos x^\circ = \frac{\text{adj}}{\text{hyp}}$	$\tan x^\circ = \frac{\text{opp}}{\text{adj}}$
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They must be remembered. You may find it helpful to use the first letter of each word:

SOH CAH TOA

It makes a nonsense word, but it seems to stick in most people’s memory!

5.1.3 Using a Calculator

To make use of the ratios, you must have another way of finding them. You can use a scientific calculator.

On some calculators, you enter the angle first and then the ratio. On others, you enter the ratio first. Check your own calculator.

eg $\sin 65.5^\circ$ Enter 65.5 then press **sin**,
or press **sin** then enter 65.5.

Answer: 0.909 96...

Most trig questions will require rounding. 3 sf are usually sufficient for giving a trig ratio, and 1 dp for giving an angle, but it will depend on the question. In practice, it is better to only round off the final answer (see Section 1.4).

When you know the sine value, for example, you must use the inverse sine to find the angle. You write this as:

$$\sin^{-1}x$$

On a calculator, this is usually obtained by pressing the **SHIFT** or **2nd Function** button then the **sin** button.

eg $\sin x^\circ = 0.65$ Enter 0.65 then press **shift** then **sin**,
or press **shift** then **sin** then enter 0.65.

Answer: $x^\circ = 40.541\dots^\circ$

Note: show the working as

$$\sin x^\circ = 0.65$$

$$x^\circ = \sin^{-1} 0.65 = 40.541\dots^\circ$$

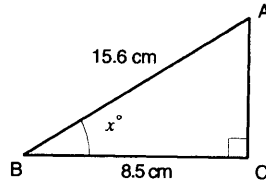
5.1.4 Using the Ratios - Finding an Angle

1. Use 2 of the sides to set up a ratio - sin, cos or tan.
2. Divide the numerator by the denominator to get a decimal value in the calculator.
3. Use the inverse trig ratio on the calculator to find the angle.

Example: In the diagram, what is the value of angle ABC, ie x° ?

AB is the hyp, BC is the adj.
Therefore use the cos ratio.

$$\begin{aligned}\cos x^\circ &= \frac{\text{adj}}{\text{hyp}} = \frac{8.5}{15.6} = 0.544\dots \\ x^\circ &= \cos^{-1} 0.544\dots \\ &= 56.98\dots\end{aligned}$$



Angle ABC is **57.0°** to 1dp.

5.1.5 Using the Ratios - Finding a Side 1 (numerator)

G

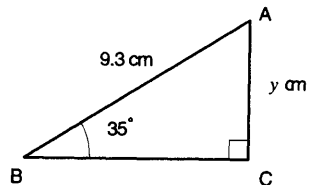
At the **G** level, the unknown side will always be in the numerator of the ratio.

1. Use 2 of the sides, including the unknown, to set up a ratio - sin, cos or tan - using the angle given.
2. Rearrange the equation to solve for the unknown.
3. Simplify.

Example: In the diagram, what is the length of AC, ie y cm?

AB is the hyp, AC is the opp.
Therefore use the sin ratio.

$$\begin{aligned}\sin 35^\circ &= \frac{\text{opp}}{\text{hyp}} = \frac{y}{9.3} \\ y &= 9.3 \times \sin 35^\circ \\ &= 5.33\dots\end{aligned}$$



Side AC is **5.3 cm** to 1dp.

5.1.6 Using the Ratios - Finding a Side 2 (denominator)

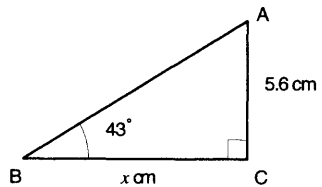
At the **C/G** level, the unknown side can be in the denominator of the ratio. This leads to a more complicated rearrangement with the trig ratio ending up in the denominator.

1. Use 2 of the sides, including the unknown, to set up a ratio - sin, cos or tan - using the angle given.
2. Rearrange the equation to solve for the unknown.
3. Simplify.

Example: In the diagram, what is the length of BC, ie x cm?

AC is the opp, BC is the adj.
Use the tan ratio therefore.

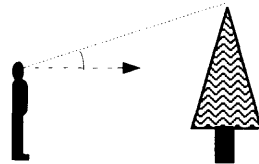
$$\begin{aligned}\tan 43^\circ &= \frac{\text{opp}}{\text{adj}} = \frac{5.6}{x} \\ x \tan 43^\circ &= 5.6 \\ x &= \frac{5.6}{\tan 43^\circ} \\ &= 6.00..\end{aligned}$$



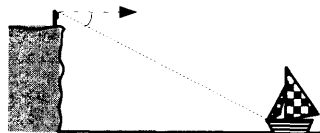
Side BC is **6.0 cm** to 1dp.

5.2 ANGLES OF ELEVATION AND DEPRESSION

The angle measured up from the horizontal to an object is called the **angle of elevation** of the object from that point.



The angle measured down from the horizontal to an object is called the **angle of depression** of the object from that point.



In small heights, it may be necessary to take the eye level of the person into account to get an accurate value. In larger measurements, it will not make a significant difference.